Integration By Parts

Idea: From the product rule, we know:

$$(uv)' = u'v + v'u$$

In terms of differentials, this is

$$d(uv) = (du)v + (dv)u$$

Integrating,

$$uv = \int d(uv) = \int v\,du + \int u\,dv$$

Rearranging,

$$\int u\,dv = uv - \int v\,du$$

ex: Note: Integrand is $u\,dv$

$$\int \underbrace{x}_{u} \underbrace{\sin x\,dx}_{dv} = uv - \int v\,du \quad \text{where} \quad u = x \quad du = dx$$

$$V = -\cos x \quad dv = \sin x\,dx$$

any antiderivative will do

$$= -x\cos x + \int \cos x\,dx$$

$$= \sin x - x\cos x + C$$

Here we chose $u = x$ because it disappears when we differentiate it

ex: Take $u = \ln x$ since we don't know $\int \ln x\,dx$.

$$\int x \ln x\,dx = uv - \int v\,du \quad \text{where} \quad u = \ln x \quad du = \frac{dx}{x}$$

$$v = \frac{x^2}{2} \quad dv = x\,dx$$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

ex: same problem as before

$$\int \underbrace{\ln x}_{u}\ \underbrace{dx}_{dv} = uv - \int v\,du \quad \text{where} \quad u = \ln x \quad du = \frac{dx}{x}$$

$$V = x \quad dv = dx$$

$$= x\ln x - \int dx$$

$$= x\ln x - x + C$$